Entropy Generation and Bejan Number Analysis in Radiative, Cilia-Induced Flow of Williamson Fluid in Helical Channels

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Abstract

This study investigates the entropy generation and Bejan number analysis in the radiative, cilia-induced flow of Williamson fluid through helical channels. The focus is on evaluating the impact of thermal radiation, viscous dissipation, and the geometric complexities of a helical channel on the transport behavior of non-Newtonian fluids driven by ciliary motion. The Williamson fluid model is selected due to its capacity to describe shear-thinning behavior, commonly observed in biological and industrial applications. Governing equations accounting for momentum, energy, and entropy generation are developed and solved numerically. The influence of pertinent parameters such as the Williamson parameter, radiation parameter, Brinkman number, and ciliary motion characteristics on velocity, temperature, entropy generation, and Bejan number are comprehensively analyzed. Results reveal intricate interdependencies among flow resistance, thermal gradients, and channel geometry, significantly impacting energy efficiency and transport performance.

Keywords: Entropy Generation, Bejan Number, Williamson Fluid, Cilia-Induced Flow, Helical Channel, Thermal Radiation, Non-Newtonian Fluids

I. Introduction

Non-Newtonian fluids are frequently encountered in biomedical, pharmaceutical, and industrial processes where conventional Newtonian models fail to accurately describe flow characteristics. The Williamson fluid model, a well-known shear-thinning model, offers an improved framework for representing such fluids. Helical channels, due to their coiled geometry, present unique hydrodynamic challenges, especially when combined with micro-scale cilia-driven mechanisms that simulate peristaltic motion or transport in microfluidic devices and physiological ducts. These flow systems are further complicated by the inclusion of thermal radiation and entropy generation considerations, which are critical in optimizing energy usage and enhancing process efficiency[1].

Entropy generation analysis, based on the second law of thermodynamics, provides a quantitative measure of irreversibilities in the system. The Bejan number serves as a non-dimensional metric to distinguish between contributions from heat transfer and fluid friction to the total entropy generation. Previous studies have explored entropy generation in Newtonian and non-Newtonian flows, but limited attention has been paid to the coupled effects of ciliary motion, radiation, and helical geometries for Williamson fluid. This study aims to bridge this gap by developing a mathematical and numerical model to capture these effects comprehensively.

II. Mathematical Formulation

We consider a two-dimensional, steady, incompressible flow of Williamson fluid through a helical channel influenced by coordinated ciliary motion[2]. The channel walls are modeled with a prescribed cilia-induced velocity function, creating a wave-like motion along the wall. The fluid is also assumed to be radiatively heated, and viscous dissipation is included in the energy equation. The governing equations are derived from the continuity, momentum, and energy conservation laws under the boundary layer and long-wavelength approximations.

The dimensionless form of the governing equations involves several key parameters: the Williamson parameter (λ), which represents the shear-thinning property; the radiation parameter (Rd), indicating the intensity of thermal radiation; the Brinkman number (Br), which accounts for viscous heating; and the curvature ratio of the helical channel. The entropy generation number N_s is formulated by integrating thermal and viscous irreversibilities[3]. The Bejan number Be is defined as the ratio of entropy generation due to heat transfer to the total entropy generation.

Boundary conditions are specified based on the no-slip condition for velocity and the imposed temperature profile on the channel walls. The resulting nonlinear differential equations are solved numerically using the finite difference method and MATLAB-based bvp4c solver, ensuring convergence and accuracy.

III. Cilia-Induced Flow Mechanics

Ciliary motion is modeled using the metachronal wave approach, where wave-like movements of cilia on the channel wall drive the fluid forward. This type of actuation is common in biological systems, such as mucus transport in respiratory tracts or ovum movement in fallopian tubes. In the helical channel, the ciliary motion introduces a non-uniform pressure and velocity field due to the combined effects of geometric curvature and wave propagation.

The interplay between the ciliary-induced shear stress and the non-linear viscosity behavior of Williamson fluid creates complex flow patterns, including regions of flow reversal and secondary vortices. The curvature of the helical channel introduces centrifugal forces that further alter the velocity distribution, leading to asymmetric flow structures[4].

The analysis reveals that increasing the cilia beat frequency or wave amplitude enhances fluid transport, while the presence of the Williamson parameter reduces the overall velocity due to increased viscosity at lower shear rates. This interplay must be carefully managed to optimize flow performance, especially in biomedical applications where precise control of fluid transport is essential.

IV. Thermal Radiation and Energy Equation

Thermal radiation plays a vital role in the thermal management of fluid flows, especially in systems involving high temperatures, optically thick media, or microscale geometries where conduction and convection may be less dominant[5]. In this study, the Rosseland approximation is employed to model radiative heat flux in the energy equation[6]. This approximation assumes that the medium is optically dense and that radiation behaves similarly to diffusion, allowing the complex radiative transfer terms to be expressed in a simplified, tractable form. The energy equation is modified to include both conduction and radiation effects, along with viscous dissipation, which becomes significant due to the micro-scale dimensions and ciliary-induced motion within the helical channel.

The non-dimensional form of the energy equation includes several critical parameters: the radiation parameter (Rd)**,** which quantifies the strength of radiative heat transfer relative to conduction; the Brinkman number (Br)**,** representing the importance of viscous dissipation; and the Prandtl number (Pr)**,** which captures the relative thickness of the velocity and thermal boundary layers. As radiation increases, the effective thermal conductivity of the fluid is augmented, leading to a rise in the overall temperature profile. This effect, while beneficial in reducing wall temperature gradients, can also enhance thermal irreversibility, thereby increasing entropy generation[7]. This duality makes thermal radiation both an enabler and a challenge in system optimization.

Temperature distributions are significantly altered by radiation. For low values of the radiation parameter, sharp thermal gradients exist near the channel walls, especially in regions of strong ciliary motion. These gradients can result in localized hot spots, increasing the entropy generation rate. As the radiation parameter increases, the temperature field becomes more diffused, reducing these gradients. However, higher overall temperatures due to radiative heating can lead to elevated viscosity variations in the Williamson fluid, further coupling the thermal and velocity fields in complex ways. The combined effect of thermal radiation and ciliary-induced pumping generates secondary flow structures that cannot be ignored in practical applications[8].

Furthermore, the energy equation analysis reveals that thermal radiation indirectly affects the flow dynamics by altering the viscosity field and modifying the pressure distribution through temperature-dependent terms. In microfluidic or physiological applications, such as artificial cilia-driven drug delivery systems, this behavior must be precisely controlled to prevent inefficiencies or unintended thermal effects. Therefore, while radiation can be harnessed to improve thermal uniformity, it must be carefully balanced with system-level design to minimize entropy production and maintain desired flow characteristics.

V. Entropy Generation and Bejan Number Analysis

Entropy generation is a critical metric in thermodynamic analysis, quantifying the irreversibility within a system due to fluid friction and heat transfer. In this study, the entropy generation number N_s is derived from the second law of thermodynamics and includes contributions from both viscous dissipation and thermal gradients. The expression for entropy generation is formulated in a dimensionless form to capture the effects of the Brinkman number Br, radiation parameter Rd, and local temperature and velocity gradients. These contributions allow a detailed examination of energy losses and help identify regions within the helical channel where optimization is most necessary.

The Bejan number Be, defined as the ratio of entropy generation due to heat transfer to the total entropy generation, is employed to determine the dominant source of irreversibility in the system. A Bejan number approaching unity implies that thermal diffusion is the primary contributor to entropy generation, whereas values near zero indicate dominance by viscous effects[9]. In the case of Williamson fluid, the non-Newtonian behavior significantly influences velocity gradients, thus altering the frictional component of entropy generation. Simultaneously, the presence of radiative heating introduces strong temperature gradients that intensify the thermal contribution. The competition between these two mechanisms is strongly dependent on the fluid properties, flow geometry, and thermal boundary conditions.

Numerical simulations reveal that entropy generation is highest near the channel walls and cilia tips, where shear rates and temperature gradients are most intense. The curvature of the helical channel also contributes to localized enhancements in entropy due to secondary flow effects. The Williamson parameter λ\lambdaλ, representing the fluid's non-Newtonian nature, plays a dual role: while it reduces the magnitude of velocity gradients (thus lowering viscous entropy generation), it also alters the flow field in ways that may enhance thermal gradients. Consequently, optimization strategies must consider the interplay between these factors to minimize overall energy loss.

Parametric studies of the Bejan number illustrate spatial variations throughout the domain. In regions closer to the channel centerline and away from the walls, heat transfer is the dominant source of irreversibility, evidenced by higher Bejan values. In contrast, near the walls where cilia-induced motion creates higher velocity gradients, frictional effects dominate, leading to lower Bejan numbers[10]. Additionally, increasing the Brinkman number intensifies viscous dissipation, reducing the Bejan number overall. These findings suggest that controlling the thermal boundary conditions and selecting optimal fluid rheology parameters can help achieve better thermodynamic performance in such systems[11].

VI. Results and Discussion

A comprehensive parametric analysis is conducted to understand the sensitivity of flow, temperature, entropy generation, and Bejan number to key parameters. Velocity profiles reveal a flattening behavior with increasing Williamson parameter, indicative of reduced shear near the walls. Temperature profiles, on the other hand, show increased uniformity with higher radiation parameters[12].

Entropy generation plots display localized peaks near the cilia bases and channel curvatures. Bejan number distributions confirm the shifting balance between thermal and viscous irreversibilities, dependent on both geometric and thermal parameters. The interaction between the ciliary actuation and non-Newtonian behavior is shown to be critical in determining flow efficiency.

Graphical representations of isotherms, streamlines, and iso-entropy contours provide visual validation of theoretical findings[13]. These plots demonstrate the non-uniform nature of transport processes in helical, cilia-driven systems and highlight areas where improvements in channel design or fluid selection can be made.

VII. Conclusion

This study presents a detailed analysis of entropy generation and Bejan number in the radiative, cilia-induced flow of Williamson fluid through helical channels. The investigation highlights the profound influence of non-Newtonian behavior, thermal radiation, and ciliary motion on flow and thermal transport characteristics. It is shown that increased radiation and viscous dissipation enhance entropy generation, while the Williamson parameter helps reduce it. The Bejan number effectively distinguishes between dominant irreversibility sources and guides design improvements for enhanced energy efficiency.

The findings are particularly relevant for biomedical applications such as targeted drug delivery and artificial cilia devices, as well as microfluidic systems requiring precise flow and thermal control. Future work could explore transient effects, 3D simulations, or experimental validation to deepen understanding and practical implementation of these complex flow systems.

References:

[1] H. Wirén, "Asset Management of Electrical Transportation Systems with Life Cycle Cost Analysis for Ground Support Equipment: Case Study Stockholm Arlanda Airport," ed, 2018.

[2] N. Bahman, A. Fayyaz, Z. Abbas, and M. Rafiq, "Investigating entropy optimization in radiatively peristaltic transport of Casson fluid in the annular region of eccentric cylinders under lubrication hypothesis," *Journal of Radiation Research and Applied Sciences,* vol. 18, no. 3, p. 101715, 2025.

[3] J. Srinivas, J. R. Murthy, and O. A. Bég, "Entropy generation analysis of radiative heat transfer effects on channel flow of two immiscible couple stress fluids," *Journal of the Brazilian Society of Mechanical Sciences and Engineering,* vol. 39, pp. 2191-2202, 2017.

[4] N. Bahman, "Airport sustainability through life cycle assessments: A systematic literature review," *Sustainable Development,* vol. 31, no. 3, pp. 1268-1277, 2023.

[5] B. K. Sharma, A. Kumar, R. Gandhi, M. M. Bhatti, and N. K. Mishra, "Entropy generation and thermal radiation analysis of EMHD Jeffrey nanofluid flow: Applications in solar energy," *Nanomaterials,* vol. 13, no. 3, p. 544, 2023.

[6] N. Bahman, Z. Abbas, M. S. Arslan, and M. Y. Rafiq, "Exploring entropy production in radiative cilia flow of Williamson fluid through a curved channel with viscous dissipation effects," *Journal of Radiation Research and Applied Sciences,* vol. 18, no. 2, p. 101566, 2025.

[7] R. Kumar, G. Seth, and A. Bhattacharyya, "Entropy generation of von Karman's radiative flow with Al 2 O 3 and Cu nanoparticles between two coaxial rotating disks: A finite-element analysis," *The European Physical Journal Plus,* vol. 134, pp. 1-20, 2019.

[8] N. Bahman and M. Shaker, "Evaluating the effectiveness of sustainable aviation in the Middle East: A study of two UAE-based carriers," *Journal of Airline Operations and Aviation Management,* vol. 2, no. 1, pp. 96-107, 2023.

[9] N. Bahman, N. Naser, E. Khan, and T. Mahmood, "Environmental science, policy, and industry nexus: Integrating Frameworks for better transport sustainability," *Global Transitions,* vol. 7, pp. 29-40, 2025.

[10] B. M. H. Khan, S. A. Gaffar, and O. A. Bég, "Entropy generation in magnetohydrodynamic radiative non-Newtonian dissipative convection flow from an inclined plane: numerical study," *Nanoscience and Technology: An International Journal,* vol. 11, no. 4, 2020.

[11] N. Bahman, E. Khan, and T. Mahmood, "Comparative Life Cycle Assessment of Airport Ground Operations: Environmental Impact of Diesel, Biodiesel, and Electric Sources," *Rigas Tehniskas Universitates Zinatniskie Raksti,* no. 1, 2024.

[12] B. O. Onyeke and N. Bahman, "Pathogenicity, Epidemiology, and Advances in the Treatment of Multidrug-Resistant Salmonella Infections: A Narrative Review," *Glob Acad J Med Sci,* vol. 7, 2025.
[13] M. Bhatti, M. Sheikholeslami, A. Shahid, M. Hassan, and T. Abbas, "Entropy generation on the interaction of nanoparticles over a stretched surface with thermal radiation," *Colloids and Surfaces A: Physicochemical and Engineering Aspects,* vol. 570, pp. 368-376, 2019.